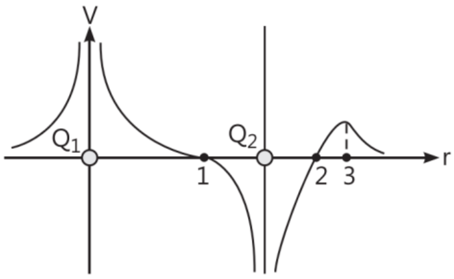
Electrostatic Potential and Capacitance

Case Study Based Questions

Case Study 1

The potential at any observation point P of a static electric field is defined as the work done by the external agent (or negative of work done by electrostatic field) in slowly bringing a unit positive point charge from infinity to the observation point. Figure shows the potential variation along the line of charges. Two point charges Q_1 and Q_2 lie along a line at a distance from each other.



Read the given passage carefully and give the answer of the following questions:

Q1. At which of the points 1, 2 and 3 is the electric field zero ?

a. 1 b. 2

c. 3 d. Both a. and b.

Q 2. The signs of charges Q_1 and Q_2 respectively are:

a. positive and negative b. negative and positive

c. positive and positive d. negative and negative

Q 3. Which of the two charges Q_1 and Q_2 is greater in magnitude?

a. Q_2 b. Q_1

c. Same d. Can't be determined

Q4. Which of the following statement is not true?

a. Electrostatic force is a conservative force.

b. Potential energy of charge q at a point is the work done per unit charge in bringing a charge from any point to infinity.

c. When two like charges lie infinite distance apart, their potential energy is zero.

d. Both a. and c.

Q5. Positive and negative charges of equal magnitude are kept at

$\left(0,0,\frac{a}{2}\right)$ and $\left(0,0,\frac{-a}{2}\right)$ respectively.

The work done by the electric field when another positive point charge is moved from (-a, 0, 0) to (0, a, 0), is:

a. positive

b. negative

c. zero

d. depends on the path connecting the initial and final positions.

Solutions

1. (c) 3

As $\frac{-dV}{dr} = E_r$, the negative of the slope of V *versus* r curve represents the components of electric field along r. Slope of curve is zero only at point 3. Therefore, the electric field vector is zero at point 3.

2. (a) positive and negative

Near positive charge, net potential is positive and near a negative charge, net potential is negative. Thus, charge Q_1 is positive and Q_2 is negative.

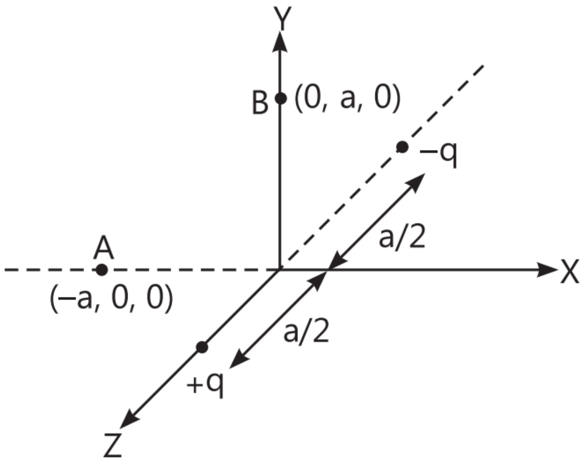
3. (b) Q_1

From the figure, it can be seen that net potential due to two charges is positive everywhere in the region left to charge Q_1. Therefore, the magnitude of potential due to charge Q_1 is greater than due to Q_2.

4. (b) Potential energy of charge q at a point is the work done per unit charge in bringing a charge from any point to infinity.

5. (c) zero

It can be seen that potential at the points both A and B are zero. When the charge is moved from A to B, work done by the electric field on the charge will be zero.



Case Study 2

This energy possessed by a system of charges by virtue of their positions. When two like charges lie infinite distance apart, their potential energy is zero because no work has to be done in moving one charge at infinite distance from the other. In carrying a charge q from point A to point B, work done $W = q(V_A - V_B)$. This work may appear as change in KE/PE of the charge. The potential energy of two charges q_1 and q_2 at a distance r in air is $\frac{q_1 q_2}{4\pi\varepsilon_0 r}$. It is measured in joule. It may be positive, negative or zero depending on the signs of q_1 and q_2.

Read the given passage carefully and give the answer of the following questions:

Q 1. Calculate work done in separating two electrons from a distance of 1 m to 2 m in air, where *e* is electric charge and *K* is electrostatic force constant.

a. Ke^2 b. $e^2/2$ c. $-Ke^2/2$ d. zero

Q 2. Two points *A* and *B* are located in diametrically opposite directions of a point charge +2 μC at distances 2 m and 1 m respectively from it. The potential difference between *A* and *B* is:

a. 3×10^3 V b. 6×10^4 V

c. -9×10^3 V d. -3×10^3 V

Q 3. Two points charges $A = +3$ nC and $B = +1$ nC are placed 5cm apart in air. The work done to move charge B towards A by 1 cm is:

a. 2.0×10^{-7} J
b. 1.35×10^{-7} J
c. 2.7×10^{-7} J
d. 12.1×10^{-7} J

Q 4. A charge Q is placed at the origin. The electric potential due to this charge at a given point in space is V. The work done by an external force in bringing another charge q from infinity up to the point is:

a. $\frac{V}{q}$
b. Vq
c. $V + q$
d. V

Solutions

1. (c) $-Ke^2/2$

$W = (\text{PE})_{\text{final}} - (\text{PE})_{\text{initial}}$

$$= \frac{Ke^2}{2} - \frac{Ke^2}{1} = \frac{-Ke^2}{2}$$

2. (c) -9×10^3 V

Here, $q = 2\mu\text{C} = 2 \times 10^{-6}\text{C}$, $r_A = 2$ m, $r_B = 1$ m

$$\therefore V_A - V_B = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$$

$$= 2 \times 10^{-6} \times 9 \times 10^9 \left[\frac{1}{2} - \frac{1}{1}\right] \text{V}$$

$$= -9 \times 10^3 \text{ V}$$

3. (b) 1.35×10^{-7}J

Given that,

$A = +3 \text{ nC} = 3 \times 10^{-9}$ C

$B = +1 \text{ nC} = 1 \times 10^{-9}$ C

Distance $r_1 = 5 \text{ cm} = 5 \times 10^{-2}$ m

and $r_2 = r_1 - 1$

$= 5 - 1 = 4 \text{ cm} = 4 \times 10^{-2}$ m

∴ Required work done = Change in potential energy of the system

$$W = U_f - U_i = K\frac{q_1q_2}{r_f} - K\frac{q_1q_2}{r_i}$$

$$= Kq_1q_2\left[\frac{1}{r_f} - \frac{1}{r_i}\right]$$

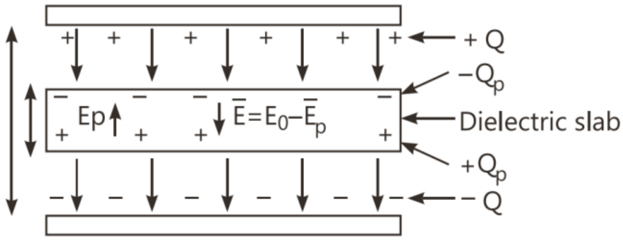
$$\therefore\ W = (9\times10^9)(3\times10^{-9}\times1\times10^{-9})\times\left[\frac{1}{4\times10^{-2}} - \frac{1}{5\times10^{-2}}\right]$$

$$= 27\times10^{-9}\times\frac{1}{20\times10^{-2}} = 1.35\times10^{-7}\text{J}$$

4. (b) Vq

Case Study 3

A dielectric slab is a substance which does not allow the flow of charges through it but permits them to exert electrostatic forces on one another. When a dielectric slab is placed between the plates, the field E_0 polarises the dielectric. This induce charge $-Q_p$ on the upper surface are $+Q_P$ on the lower surface of the dielectric. These induced charges set up a field E_p inside the dielectric in the opposite direction of $\vec{E}_0$ as shown.



Read the given passage carefully and give the answer of the following questions:

Q 1. In a parallel plate capacitor, the capacitance increases from 4 μF to 80 μF, on introducing a dielectric medium between the plates. What is the dielectric constant of the medium?

a. 10 b. 20 c. 50 d. 100

Q 2. A parallel plate capacitor with air between the plates has a capacitance of 8 pF. The separation between the plates is now reduced half and the space between them is filled with a medium of dielectric constant 5. Calculate the value of capacitance of the capacitor in second case.

a. 8 pF b. 10 pF c. 80 pF d. 100 pF

Q 3. A dielectric introduced between the plates of a parallel plate condenser:

a. decreases the electric field between the plates
b. increases the capacity of the condenser
c. increases the charge stored in the condenser
d. increases the capacity of the condenser

Q 4. A parallel plate capacitor of capacitance 1 pF has separation between the plates *d*. When the distance of separation becomes 2*d* and wax of dielectric constant *x* is inserted in it, the capacitance becomes 2 pF. What is the value of *x*?

a. 2 b. 4 c. 6 d. 8

Q 5. A parallel plate capacitor having area *A* and separated by distance *d* is filled by copper plate of thickness *b*. The new capacity is:

a. $\frac{\varepsilon_0 A}{d+\frac{b}{2}}$ b. $\frac{\varepsilon_0 A}{2d}$ c. $\frac{\varepsilon_0 A}{d-b}$ d. $\frac{2\varepsilon_0 A}{d+\frac{b}{2}}$

Solutions

1. (b) 20

Dielectric constant

$$K = \frac{\text{Capacitance with dielectric}}{\text{Capacitance without dielectric}}$$

$$= \frac{80\mu F}{4\mu F} = 20$$

2. (c) 80 pF

Capacitance of the capacitor with air between plates

$$C' = \frac{\varepsilon_0 A}{d} = 8\text{pF}$$

With the capacitor filled with dielectric ($K = 5$) between its plates and the distance between the plates is reduced by half, capacitance become

$$C = \frac{\varepsilon_0 KA}{d/2} = \frac{\varepsilon_0 \times 5 \times A}{d/2} = 10\ C' = 10 \times 8 = 80\ \text{pF}$$

3. (d) increases the capacity of the condenser

If a dielectric medium of dielectric constant K is filled completely between the plates, then capacitance increases by K times.

4. (b) 4

$$C = \frac{\varepsilon_0 A}{d} = 1\text{pF} \quad \text{...(1)}$$

$$C' = \frac{X\varepsilon_0 A}{(2d)} = 2\ \text{pF} \quad \text{...(2)}$$

Dividing eq. (2) by eq. (1),

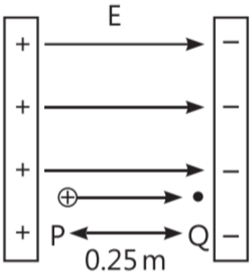
$$\frac{X}{2} = \frac{2}{1} \Rightarrow X = 4$$

5. (c) $\frac{\varepsilon_0 A}{d - b}$

As capacitance, $C_0 = \frac{\varepsilon_0 A}{d}$

$\therefore$ After inserting copper plate, $C = \frac{\varepsilon_0 A}{d - b}$

Case Study 4



Potential difference (ΔV) between two points A and B separated by a distance x in a uniform electric field E is given by $\Delta V = -Ex$, where x is measured parallel to the field lines. If a charge q_0 moves from P to Q, the change in potential energy (ΔU) is given as $\Delta U = q_0 \Delta V$. A proton is released from rest in uniform electric field of magnitude $4.0 \times 10^8 \text{ Vm}^{-1}$ directly along the positive X-axis. The proton undergoes a displacement of 0.25 m in the direction of E.

Mass of a proton $= 1.66 \times 10^{-27}$ kg and charge of proton $= 1.6 \times 10^{-19}$C

Read the given passage carefully and give the answer of the following questions:

Q 1. What will be the change in electric potential of the proton between the points *A* and *B*?

Q 2. What will the change in electric potential energy of the proton for displacement from *A* to *B*?

Q 3. Calculate the mutual electrostatic potential energy between two protons which are at a distance of 9×10^{-15} m, in $_{92}U^{235}$ nucleus.

Q 4. If a system consists of two charges 4 μC and –3 μC with no external field placed at (–5cm, 0, 0) and (5 cm, 0, 0) respectively, find the amount of work required to separate the two charges infinitely away from each other.

Q 5. As the proton moves from *P* to *Q*, then what will happen?

Solutions

1. $\Delta V = -E\,\Delta X = -(4 \times 10^8) \times 0.25 = -10^8$ V

2. As, $\Delta U = q\,\Delta V = 1.6 \times 10^{-19} \times (-1.0 \times 10^8)$

$= -1.6 \times 10^{-11}$ J

3. Potential energy, $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$

$$= \frac{9\times10^9 \times 1.6\times10^{-19} \times 1.6\times10^{-19}}{9\times10^{-15}}$$

$$= 2.56 \times 10^{-14}\,\text{J}$$

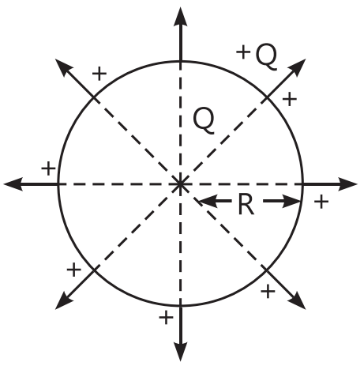
4. $$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$$

$$= \frac{9\times10^9 \times 4\times10^{-6} \times (-3)\times10^{-6}}{0.1} = -1.1\text{ J}$$

5. As proton moves in the direction of the electric field or from P to Q, then its potential energy decreases.

Case Study 5

The electrical capacitance of a conductor is the measure of its ability to hold electric charge.



An isolated spherical conductor of radius R and charge Q is uniformly distributed over its entire surface. It can be assumed to be concentrated at the centre of the sphere. The potential at any point on the surface of the spherical conductor will be

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

Capacitance of the spherical conductor situated in vacuum is

$$C = \frac{Q}{V} = \frac{Q}{\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R}} \quad \text{or } C = 4\pi\varepsilon_0 R$$

Clearly, the capacitance of a spherical conductor is proportional to its radius.

The radius of the spherical conductor is 1F capacitance is $R = \frac{1}{4\pi\varepsilon_0}.C$ and this radius is about 1500 times the radius of earth ($\sim 6 \times 10^3$ km).

Read the given passage carefully and give the answer of the following questions:

Q1. If an isolated sphere has a capacitance 50 pF, then what is the radius sphere?

Q2. How much charge should be placed on a capacitance of 25 pF to raise its potential to 10^5 V?

Q3. Metallic sphere of radius R is charged to potential V. Then charge q is proportional to which parameter?

Q4. If 64 identical spheres of charge q and capacitance C each are combined to form a large sphere, what will be the charge and capacitance of the large sphere?

Solutions

1. Here, $C = 50$ pF $= 50 \times 10^{-12}$ F.

$$R = \frac{1}{4\pi\varepsilon_0}.C = 9 \times 10^9 \text{mF}^{-1} \times 50 \times 10^{-12} \text{ F}$$

$$= 45 \times 10^{-2} \text{m} = 45 \text{ cm}$$

2. As, $q = CV = 25 \times 10^{-12} \times 10^5 = 2.5 \mu C$

3. As charge, $q = CV = (4\pi\varepsilon_0 R)V$

$\therefore$ q depends on both V and R, *i.e.*, q is proportional to both R and V.

4. 64 drops have formed a single drop of radius R.

Volume of large sphere = 64 × Volume of small sphere

$$\therefore \quad \frac{4}{3}\pi R^3 = 64 \times \frac{4}{3}\pi r^3$$

$$\Rightarrow \quad R = 4r \quad \text{and} \quad Q_{total} = 64q$$

$$C' = 4\pi\varepsilon_0 R \quad \Rightarrow \quad C' = (4\pi\varepsilon_0) \cdot 4r$$

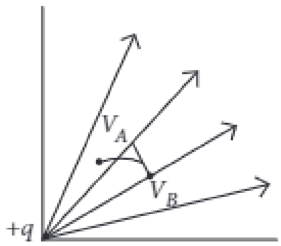
$$\Rightarrow \quad C' = 4C$$

Solutions for Questions 6 to 15 are Given Below

Case Study 6

Electrostatics Potential and Potential Energy

Electrostatic potential energy of a system of point charges is defined as the total amount of work done in bringing the different charges to their respective positions from infinitely charge mutual separations. The work is stored in the system of two point charges in the form of electrostatic potential energy U of the system. Electric potential difference between any points A and B in an electric field is the amount of work done in moving a unit positive test charge from A to B along any path agents the electrostatic force $V_B - V_A = \frac{W_{AB}}{q_o} = \int \vec{E}.dl$.



(i) A test charge is moved from lower potential point to a higher potential point. The potential energy of test charge will

(a) remain the same (b) increase

(c) decrease (d) become zero

(ii) Which of the following statement is not true?

(a) Electrostatic force is a conservative force.

(b) Potential energy of charge q at a point is the work done per unit charge in bringing a charge from any point to infinity.

(c) Spring force and gravitational force are conservative force.

(d) Both (a) and (c).

(iii) Work done in moving a charge from one point to another inside a uniformly charged conducting sphere is

(a) always zero (b) non-zero (c) may be zero (d) none of these

(iv) The work done in bringing a unit positive charge from infinite distance to a point at distance x from a positive charge Q is W. Then the potential ϕ at that point is

(a) $\frac{WQ}{x}$ (b) W (c) $\frac{W}{x}$ (d) WQ

(v) If 1μ C charge is shifted from A to B and it is found that work done by an external force is 40μ J. In doing so against electrostatics force, the potential difference $V_A - V_B$ is

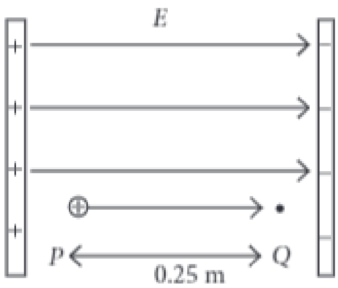
(a) 40 V (b) –40 V (c) 20 V (d) –60 V

Case Study 7

Potential Energy of the Proton

Potential difference (ΔV) between two points A and B separated by a distance x, in a uniform electric field E is given by $\Delta V = -Ex$, where x is measured parallel to the field lines. If a charge q_0 moves from P to Q, the change in potential energy (ΔU) is given as $\Delta U = q_0 \Delta V$. A proton is released from rest in uniform electric field of magnitude 4.0×10^8 V m^{-1} directed along the positive X-axis. The proton undergoes a displacement of 0.25 m in the direction of E.

Mass of a proton = 1.66×10^{-27} kg and charge of proton = 1.6×10^{-19} C



(i) The change in electric potential of the proton between the points A and B is

(a) -1×10^8 V (b) 1×10^8 V

(c) 6.4×10^{-19} V (d) -6.4×10^{-19} V

(ii) The change in electric potential energy of the proton for displacement from A to B is

(a) 1.6×10^{11} J (b) 0.5×10^{23} J

(c) -1.6×10^{-11} J (d) 3.2×10^{22} J

(iii) The mutual electrostatic potential energy between two protons which are at a distance of 9×10^{-15} m, in $_{92}U^{235}$ nucleus is

(a) 1.56×10^{-14} J (b) 5.5×10^{-14} J

(c) 2.56×10^{-14} J (d) 4.56×10^{-14} J

(iv) If a system consists of two charges 4 mC and –3 mC with no external field placed at (–5 cm, 0, 0) and (5 cm, 0, 0) respectively. The amount of work required to separate the two charges infinitely away from each other is

(a) –1.1 J (b) 2 J

(c) 2.5 J (d) 3 J

(v) As the proton moves from P to Q, then

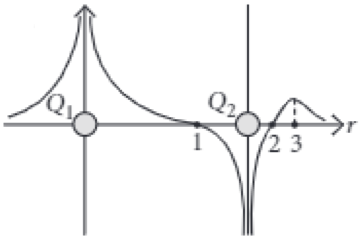
(a) the potential energy of proton decreases (b) the potential energy of proton increases

(c) the proton loses kinetic energy (d) total energy of the proton increases.

Case Study 8

Potential of Two Point Charges

The potential at any observation point P of a static electric field is defined as the work done by the external agent (or negative of work done by electrostatic field) in slowly bringing a unit positive point charge from infinity to the observation point. Figure shows the potential variation along the line of charges. Two point charges Q_1 and Q_2 lie along a line at a distance from each other.



(i) At which of the points 1, 2 and 3 is the electric field is zero?
(a) 1 (b) 2 (c) 3 (d) Both (a) and (b)

(ii) The signs of charges Q_1 and Q_2 respectively are
(a) positive and negative (b) negative and positive
(c) positive and positive (d) negative and negative

(iii) Which of the two charges Q_1 and Q_2 is greater in magnitude?
(a) Q_2 (b) Q_1 (c) Same (d) Can't determined

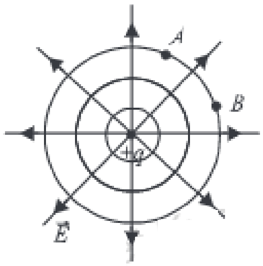
(iv) Which of the following statement is not true?
(a) Electrostatic force is a conservative force.
(b) Potential energy of charge q at a point is the work done per unit charge in bringing a charge from any point to infinity.
(c) When two like charges lie infinite distance apart, their potential energy is zero.
(d) Both (a) and (c).

(v) Positive and negative point charges of equal magnitude are kept at $\left(0, 0, \frac{a}{2}\right)$ and $\left(0, 0, \frac{-a}{2}\right)$ respectively.
The work done by the electric field when another positive point charge is moved from $(-a, 0, 0)$ to $(0, a, 0)$ is
(a) positive
(b) negative
(c) zero
(d) depends on the path connecting the initial and final positions.

Case Study 9

Equipotential Surfaces

For the various charge systems, we represent equipotential surfaces by curves and line of force by full line curves. Between any two adjacent equipotential surfaces, we assume a constant potential difference the equipotential surfaces of a single point charge are concentric spherical shells with their centres at the point charge. As the lines of force point radially outwards, so they are perpendicular to the equipotential surfaces at all points.



(i) Identify the wrong statement.
(a) Equipotential surface due to a single point charge is spherical.
(b) Equipotential surface can be constructed for dipoles too.
(c) The electric field is normal to the equipotential surface through the point.
(d) The work done to move a test charge on the equipotential surface is positive.

(ii) Nature of equipotential surface for a point charge is
(a) Ellipsoid with charge at foci
(b) Sphere with charge at the centre of the sphere
(c) Sphere with charge on the surface of the sphere
(d) Plane with charge on the surface.

(iii) A spherical equipotential surface is not possible
(a) inside a uniformly charged sphere
(b) for a dipole
(c) inside a spherical condenser
(d) for a point charge

(iv) The work done in carrying a charge q once round a circle of radius a with a charge Q at its centre is
(a) $\frac{qQ}{4\pi\varepsilon_0 a}$
(b) $\frac{qQ}{4\pi\varepsilon_0 a^2}$
(c) $\frac{q}{4\pi\varepsilon_0 a}$
(d) zero

(v) The work done to move a unit charge along an equipotential surface from P to Q
(a) must be defined as $-\int_P^Q \vec{E}.d\vec{l}$
(b) is zero
(c) can have a non-zero value
(d) both (a) and (b) are correct.

Case Study 10

Electric Potential Energy

This energy possessed by a system of charges by virtue of their positions. When two like charges lie infinite distance apart, their potential energy is zero because no work has to be done in moving one charge at infinite distance from the other.

In carrying a charge q from point A to point B, work done $W = q(V_A - V_B)$. This work may appear as change in KE/PE of the charge. The potential energy of two charges q_1 and q_2 at a distance r in air is $\frac{q_1 q_2}{4\pi\varepsilon_0 r}$. It is measured in joule. It may be positive, negative or zero depending on the signs of q_1 and q_2.

(i) Calculate work done in separating two electrons form a distance of 1 m to 2 m in air, where e is electric charge and k is electrostatic force constant.
(a) ke^2
(b) $e^2/2$
(c) $-ke^2/2$
(d) zero

(ii) Four equal charges q each are placed at four corners of a square of side a each. Work done in carrying a charge $-q$ from its centre to infinity is
(a) zero
(b) $\frac{\sqrt{2}q^2}{\pi\varepsilon_0 a}$
(c) $\frac{\sqrt{2}q}{\pi\varepsilon_0 a}$
(d) $\frac{q^2}{\pi\varepsilon_0 a}$

(iii) Two points A and B are located in diametrically opposite directions of a point charge of +2 μC at distances 2 m and 1 m respectively from it. The potential difference between A and B is

(a) 3×10^3 V (b) 6×10^4 V (c) -9×10^3 V (d) -3×10^3 V

(iv) Two point charges A = +3 nC and B = +1 nC are placed 5 cm apart in air. The work done to move charge B towards A by 1 cm is

(a) 2.0×10^{-7} J (b) 1.35×10^{-7} J (c) 2.7×10^{-7} J (d) 12.1×10^{-7} J

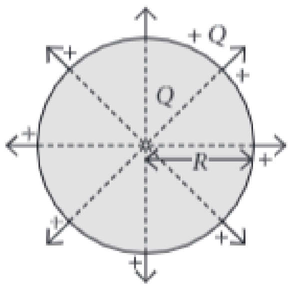
(v) A charge Q is placed at the origin. The electric potential due to this charge at a given point in space is V. The work done by an external force in bringing another charge q from infinity up to the point is

(a) $\frac{V}{q}$ (b) Vq (c) $V + q$ (d) V

Case Study 11

Spherical Capacitor

The electrical capacitance of a conductor is the measure of its ability to hold electric charge.
An isolated spherical conductor of radius R. The charge Q is uniformly distributed over its entire surface. It can be assumed to be concentrated at the centre of the sphere. The potential at any point on the surface of the spherical conductor will be $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$.



Capacitance of the spherical conductor situated in vacuum is $C = \frac{Q}{V} = \frac{Q}{\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{R}}$ or $C = 4\pi\varepsilon_0 R$

Clearly, the capacitance of a spherical conductor is proportional to its radius.

The radius of the spherical conductor of 1F capacitance is $R = \frac{1}{4\pi\varepsilon_0}$. C and this radius is about 1500 times the radius of the earth ($\sim 6 \times 10^3$ km).

(i) If an isolated sphere has a capacitance 50pF. Then radius is

(a) 90 cm (b) 45 cm (c) 45 m (d) 90 m

(ii) How much charge should be placed on a capacitance of 25 pF to raise its potential to 10^5 V?

(a) 1 μC (b) 1.5 μC (c) 2 μC (d) 2.5 μC

(iii) Dimensions of capacitance is

(a) $[M L^{-2} T^4 A^2]$ (b) $[M^{-1} L^{-1} T^3 A^1]$ (c) $[M^{-} L^{-2} T^4 A^2]$ (d) $[M^0 L^{-2} T^4 A^1]$

(iv) Metallic sphere of radius R is charged to potential V. Then charge q is proportional to

(a) V (b) R (c) both V and R (d) none of these.

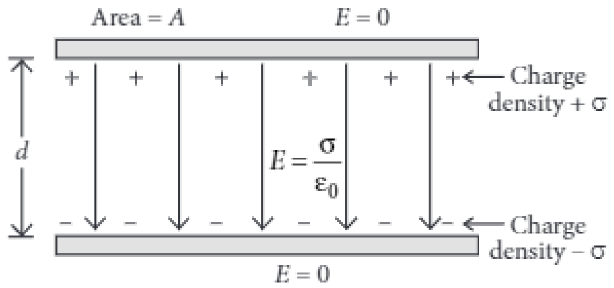
(v) If 64 identical spheres of charge q and capacitance C each are combined to form a large sphere. The charge and capacitance of the large sphere is

(a) $64q, C$ (b) $16q, 4C$ (c) $64q, 4C$ (d) $16q, 64C$

Case Study 12

Parallel Plate Capacitor

The simplest and the most widely used capacitor is the parallel plate capacitor. It consists of two large plane parallel conducting plates, separated by a small distance.
In the outer regions above the upper plate and below the lower plate, the electric fields due to the two charged plates cancel out. The net field is zero.
In the inner region between the two capacitor plates, the electric fields due to the two charged plates add up. The net field is $\frac{\sigma}{\varepsilon_0}$.



For a uniform electric field, potential difference between the plates = Electric field × distance between the plates. Capacitance of the parallel plate capacitor is, the charge required to supplied to either of the conductors of the capacitor so as to increase the potential difference between then by unit amount.

(i) A parallel plate capacitor is charged and then isolated. The effect of increasing the plate separation on charge, potential and capacitance respectively are

(ii) In a parallel plate capacitor, the capacity increases if

(a) area of the plate is decreases (b) distance between the plates increases
(c) area of the plate is increases (d) dielectric constant decreases.

(iii) A parallel plate capacitor has two square plates with equal and opposite charges. The surface charge densities on the plates are $+\sigma$ and $-\sigma$ respectively. In the region between the plates the magnitude of the electric field is

(a) $\frac{\sigma}{2\varepsilon_0}$ (b) $\frac{\sigma}{\varepsilon_0}$ (c) 0 (d) none of these.

(iv) If a parallel plate air capacitor consists of two circular plates of diameter 8 cm. At what distance should the plates be held so as to have the same capacitance as that of sphere of diameter 20 cm?

(a) 9 mm (b) 4 mm (c) 8 mm (d) 2 mm

(v) If a charge of $+2.0 \times 10^{-8}$ C is placed on the positive plate and a charge of -1.0×10^{-8} C on the negative plate of a parallel plate capacitor of capacitance 1.2×10^{-3} μF, then the potential difference developed between the plates is

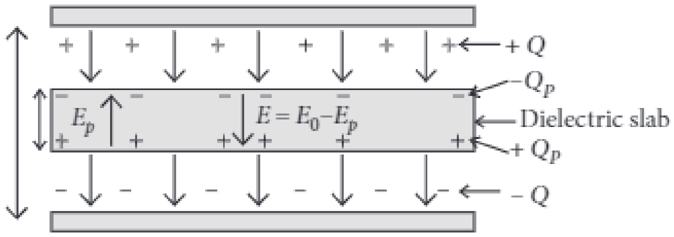
(a) 6.25 V (b) 3.0 V (c) 12.5 V (d) 25 V

Case Study 13

Dielectric Slab

A dielectric slab is a substance which does not allow the flow of charges through it but permits them to exert electrostatic forces on one another.

When a dielectric slab is placed between the plates, the field E_0 polarises the dielectric. This induces charge $-Q_p$ on the upper surface and $+Q_p$ on the lower surface of the dielectric. These induced charges set up a field E_p inside the dielectric in the opposite direction of $\vec{E}_0$ as shown.



(i) In a parallel plate capacitor, the capacitance increases from 4μF to 80 μF, on introducing a dielectric medium between the plates. What is the dielectric constant of the medium?

(a) 10 (b) 20 (c) 50 (d) 100

(ii) A parallel plate capacitor with air between the plates has a capacitance of 8 pF. The separation between the plates is now reduced half and the space between them is filled with a medium of dielectric constant 5. Calculate the value of capacitance of the capacitor in second case.

(a) 8 pF (b) 10 pF (c) 80 pF (d) 100 pF

(iii) A dielectric introduced between the plates of a parallel plate condenser

(a) decreases the electric field between the plates
(b) increases the capacity of the condenser
(c) increases the charge stored in the condenser
(d) increases the capacity of the condenser

(iv) A parallel plate capacitor of capacitance 1 pF has separation between the plates is d. When the distance of separation becomes $2d$ and wax of dielectric constant x is inserted in it the capacitance becomes 2 pF. What is the value of x?

(a) 2 (b) 4 (c) 6 (d) 8

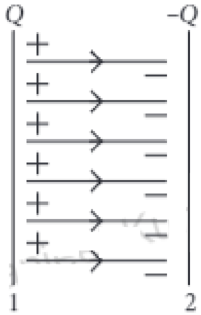
(v) A parallel plate capacitor having area A and separated by distance d is filled by copper plate of thickness b. The new capacity is

(a) $\dfrac{\varepsilon_0 A}{d+\dfrac{b}{2}}$ (b) $\dfrac{\varepsilon_0 A}{2d}$ (c) $\dfrac{\varepsilon_0 A}{d-b}$ (d) $\dfrac{2\varepsilon_0 A}{d+\dfrac{b}{2}}$

Case Study 14

Energy Stored in Capacitor

A capacitor is a device to store energy. The process of charging up a capacitor involves the transferring of electric charges from its one place to another. This work done in charging the capacitor is stored as its electrical potential energy.



If q is the charge and V is the potential difference across a capacitor at any instant during its charging, then small work done in storing an additional small charge dq against the repulsion of charge q already stored on it is $dW = V.dq = (q/C)dq$

(i) A system of 2 capacitors of capacitance 2 μF and 4 μF is connected in series across a potential difference of 6 V. The energy stored in the system is
(a) 3 μJ (b) 24 μJ (c) 30 μJ (d) 108 μJ

(ii) A capacitor of capacitance of 10 μF is charged to 10 V. The energy stored in it is
(a) 100 μJ (b) 500 μJ (c) 1000 μJ (d) 1 μJ

(iii) A parallel plate air capacitor has capacity C farad, potential V volt and energy E joule. When the gap between the plates is completely filled with dielectric
(a) both V and E increase (b) both V and E decrease
(c) V decreases, E increases (d) V increases, E decreases

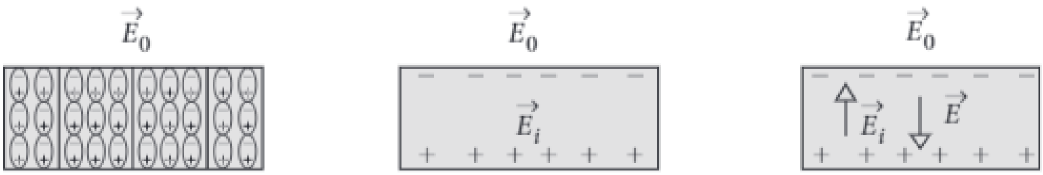
(iv) A capacitor with capacitance 5 μF is charged to 5 μC. If the plates are pulled apart to reduce the capacitance to 2 μF, how much work is done?
(a) 6.25×10^{-6} J (b) 3.75×10^{-6} J (c) 2.16×10^{-6} J (d) 2.55×10^{-6} J

(v) A metallic sphere of radius 18 cm has been given a charge of 5×10^{-6} C. The energy of the charged conductor is
(a) 0.2 J (b) 0.6 J (c) 1.2 J (d) 2.4 J

Case Study 15

Polarisation of Dielectric

When an insulator is placed in an external field, the dipoles become aligned. Induced surface charges on the insulator establish a polarization field $\vec{E}_i$ in its interior. The net field $\vec{E}$ in the insulator is the vector sum of $\vec{E}_0$ and $\vec{E}_i$ as shown in the figure.



(a) (b) (c)

On the application of external electric field, the effect of aligning the electric dipoles in the insulator is called polarisation and the field $\vec{E}_i$ is known as the polarisation field.
The dipole moment per unit volume of the dielectric is known as polarisation $(\vec{P})$.
For linear isotropic dielectrics, $\vec{P} = \chi\vec{E}$, where χ = electrical susceptibility of the dielectric medium.

(i) Which among the following is an example of polar molecule?
(a) O_2 (b) H_2 (c) N_2 (d) HCl

(ii) When air is replaced by a dielectric medium of constant K, the maximum force of attraction between two charges separated by a distance

(a) increases K times (b) remains unchanged (c) decreases K times (d) increases $2K$ times.

(iii) Which of the following is a dielectric?

(a) Copper (b) Glass (c) Antimony (Sb) (d) None of these

(iv) For a polar molecule, which of the following statements is true ?

(a) The centre of gravity of electrons and protons coincide.

(b) The centre of gravity of electrons and protons do not coincide.

(c) The charge distribution is always symmetrical.

(d) The dipole moment is always zero.

(v) When a comb rubbed with dry hair attracts pieces of paper. This is because the

(a) comb polarizes the piece of paper

(b) comb induces a net dipole moment opposite to the direction of field

(c) electric field due to the comb is uniform

(d) comb induces a net dipole moment perpendicular to the direction of field

HINTS & EXPLANATIONS

6. **(i)** **(c)**

(ii) **(b)**

(iii) **(a):** Since, $E = 0$ inside the conductor and has no tangential component on the surface, no work is done in moving a small test charge within the conductor and on its surface.

(iv) **(b):** The work done in bringing unit positive charge from infinity to a point which is at a distance x from the positive charge Q is defined as the potential at the given point due to the charge Q. Therefore $\phi = W$.

(v) **(b):** $W_{ext} = q_0 \Delta V$

$(W_{AB})_{ext} = q(V_B - V_A)$

$40\mu J = 1\mu C\,(V_B - V_A)$

$V_A - V_B = -40$ V

7. (i) (a) : As $\Delta V = -E\Delta x = -(4.0 \times 10^8 \text{ V/m})(0.25 \text{ m}) = -10^8$ V

(ii) (c) : As $\Delta U = q_0\Delta V = (1.6 \times 10^{-19}) \times (-1.0 \times 10^8 \text{ V}) = -1.6 \times 10^{-11}$ V

(iii) (c) : Here, $q_1 = q_2 = 1.6 \times 10^{-19}$ C, $r = 9 \times 10^{-15}$ m

$$U = \frac{9\times10^9 \times 1.6\times10^{-19} \times 1.6\times10^{-19}}{9\times10^{-15}} = 2.56\times10^{-14} \text{ J}$$

(iv) (a) : Here, $q_1 = 4$ μC, $q_2 = -3$ μC

$r = 10$ cm $= 0.1$ m

Electrostatic potential energy,

$$U = \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{r} = 9\times10^9 \times \frac{4\times10^{-6}\times(-3)\times10^{-6}}{0.1} = -1.1 \text{ J}$$

(v) (a) : As proton moves in the direction of the electric field, then its potential energy decreases.

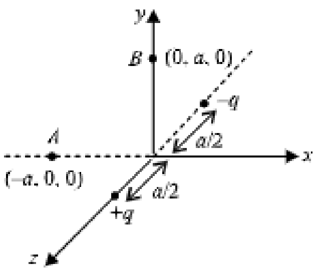
8. (i) (c) : As $\frac{-dV}{dr} = E_r$, the negative of the slope of V versus r curve represents the component of electric field along r. Slope of curve is zero only at point 3. Therefore, the electric field vector is zero at point 3.

(ii) (a) : Near positive charge, net potential is positive and near a negative charge, net potential is negative. Thus, charge Q_1 is positive and Q_2 is negative.

(iii) (b) : From the figure, it can be seen that net potential due to two charges is positive everywhere in the region left to charge Q_1. Therefore the magnitude of potential due to charge Q_1 is greater than due to Q_2.

(iv) (b)

(v) (c) : It can be seen that potential at the points both A and B are zero. When the charge is moved from A to B, work done by the electric field on the charge will be zero.

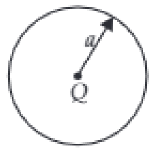


9. (i) (d)

(ii) (b)

(iii) (b)

(iv) (d) : The electrical potential at any point on circle of radius a due to charge Q at its centre is $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{a}$



It is an equipotential surface.

Hence, work done in carrying a charge q round the circle is zero.

(v) (d) : Work done to move a unit charge along an equipotential surface from P to Q,

$$W = -\int_P^Q \vec{E}.d\vec{l}$$

On equipotential surface $\vec{E} \perp d\vec{l}$

$$W = -\int_P^Q E(dl)\cos 90° = 0$$

10. (i) (c) : $W = (\text{P.E.})_{\text{final}} - (\text{P.E.})_{\text{initial}}$

$$= \frac{ke^2}{2} - \frac{ke^2}{1} = \frac{-ke^2}{2}$$

(ii) (b) : Potential at the centre of the square due to four equal charges q at four corners

$$V = \frac{4q}{4\pi\varepsilon_0(a\sqrt{2})/2} = \frac{\sqrt{2}q}{\pi\varepsilon_0 a}$$

$$W_{0\to\infty} = -W_{\infty\to0} = -(-q)V = \frac{\sqrt{2}q^2}{\pi\varepsilon_0 a}$$

(iii) (c) : Here, $q = 2$ μC $= 2 \times 10^{-6}$ C, $r_A = 2$ m, $r_B = 1$ m

$$\therefore\ V_A - V_B = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$$

$$= 2\times10^{-6}\times9\times10^9\left[\frac{1}{2}-\frac{1}{1}\right] \text{V} = -9\times10^3 \text{ V}$$

(iv) (b) : Required work done = Change in potential energy of the system

$$W = U_f - U_i = k\frac{q_1q_2}{r_f} - k\frac{q_1q_2}{r_i} = k\,q_1q_2\left[\frac{1}{r_f}-\frac{1}{r_i}\right]$$

$$\therefore\ W = (9\times10^9)(3\times10^{-9}\times1\times10^{-9}) \times\left[\frac{1}{4\times10^{-2}} - \frac{1}{5\times10^{-2}}\right]$$

$$= 27\times10^{-7}\times(0.05) = 1.35\times10^{-7} \text{ J}$$

(v) (b)

11. (i) (b) : Here $C = 50\text{pF} = 50 \times 10^{-12}$ F, $V = 10^4$ V

$$R = \frac{1}{4\pi\varepsilon_0}\cdot C = 9\times10^9 \text{ mF}^{-1}\times 50\times10^{-12} \text{ F}$$

$= 45 \times 10^{-2}$ m $= 45$ cm

(ii) (d): As $q = CV = 25 \times 10^{-12} \times 10^5 = 2.5\ \mu C$

(iii) (c)

(iv) (c) : As charge, $q = CV = (4\pi\varepsilon_0 R)V$

$\therefore$ q depends on both V and R.

(v) (c) : 64 drops have formed a single drop of radius R.
Volume of large sphere = 64 × Volume of small sphere

$\therefore \frac{4}{3}\pi R^3 = 64\frac{4}{3}\pi r^3 \Rightarrow R = 4r$ and $Q_{total} = 64q$

$C' = 4\pi\varepsilon_0 R \Rightarrow C' = (4\pi\varepsilon_0).4r \Rightarrow C' = 4C$

12. (i) (b): As the capacitor is isolated after charging, charge Q on it remains constant. Plate separation d increases, capacitance decreases as $C = \frac{\varepsilon_0 A}{d}$ and hence, potential increases as $V = \frac{Q}{C}$.

(ii) (c): In a parallel plate capacitor, the capacity of capacitor

$C = \frac{K\varepsilon_0 A}{d}$ i.e., $C \propto A$

The capacity of capacitor increases if area of the plate increases.

(iii) (b): The magnitude of the electric field between the plates is $E = \frac{\sigma}{2\varepsilon_0} - \left(-\frac{\sigma}{2\varepsilon_0}\right) = \frac{\sigma}{\varepsilon_0}$

(iv) (b): As, $\frac{\varepsilon_0 A}{d} = 4\pi\varepsilon_0 R$ or $\frac{\varepsilon_0 \pi D^2}{4d} = 4\pi\varepsilon_0 R$

or $d = \frac{D^2}{16R} = \frac{(0.08)^2}{16 \times 0.10} = 4 \times 10^{-3}$ m = 4 mm

(v) (c): Here, $V = \frac{q_1 - q_2}{2C}$

$$= \frac{2.0\times10^{-8} + 1.0\times10^{-8}}{2\times1.2\times10^{-9}} = 12.5\ V$$

13. (i) (b): $k = \frac{\text{Capacitance with dielectric}}{\text{Capacitance without dielectric}}$

$= \frac{80\ \mu F}{4\ \mu F} = 20$

(ii) (c): Capacitance of the capacitor with air between plates

$$C' = \frac{\varepsilon_0 A}{d} = 8\ pF$$

With the capacitor is filled with dielectric (k = 5) between its plates and the distance between the plates is reduced by half, capacitance become

$$C = \frac{\varepsilon_0 kA}{d/2} = \frac{\varepsilon_0 \times 5 \times A}{d/2} = 10\ C' = 10 \times 8 = 80\ pF.$$

(iii) (d): If a dielectric medium of dielectric constant K is filled completely between the plates then capacitance increases by K times.

(iv) (b): $C = \frac{\varepsilon_0 A}{d} = 1\ pF$ ...(i)

$C' = \frac{x\varepsilon_0 A}{(2d)} = 2\ pF$ ... (ii)

Divide (ii) by (i), $x/2 = 2/1 \Rightarrow x = 4$

(v) (c) : As capacitance, $C_o = \frac{\varepsilon_o A}{d}$

$\therefore$ After inserting copper plate, $C = \frac{\varepsilon_o A}{d-b}$

14. (i) (b): As, $C_1 = 2\ \mu F$, $C_2 = 4\ \mu F$

In series combination, the equivalent capacitance will be, $C = \frac{C_1 C_2}{C_1 + C_2} = \left(\frac{2\times4}{2+4}\right)\mu F = \frac{4}{3}\mu F$

Potential difference applied, $V = 6$ V

Energy stored in the system, $U = \frac{1}{2}CV^2$

$= \frac{1}{2}\times\frac{4}{3}\times10^{-6}\times(6)^2\ J = 24\ \mu J$

(ii) (b): The energy stored in a capacitor is

$U = \frac{1}{2}CV^2 = \frac{1}{2}\times(10\times10^{-6})(10)^2 = 500\ \mu J$

(iii) (b): When the gap between the plates is completely filled with dielectric of dielectric constant K, then potential is

$$V = \frac{Qd}{A\varepsilon_0 K} \quad ...(i)$$

and electric field is

$$E = \frac{Q}{A\varepsilon_0 K} \quad ...(ii)$$

From equations (i) and (ii), both electric field and potential decrease.

(iv) (b): Work done = $U_f - U_i = \frac{1}{2}\frac{q^2}{C_f} - \frac{1}{2}\frac{q^2}{C_i}$

$= \frac{q^2}{2}\left[\frac{1}{C_f} - \frac{1}{C_i}\right] = \frac{(5\times10^{-6})^2}{2}\left[\frac{1}{2\times10^{-6}} - \frac{1}{5\times10^{-6}}\right]$

$= 3.75 \times 10^{-6}$ J.

(v) (b) : Here $r = 18$ cm $= 18 \times 10^{-2}$ m, $q = 5 \times 10^{-6}$ C

As $C = 4\pi\varepsilon_0 r = \dfrac{18 \times 10^{-2}}{9 \times 10^9} = 2 \times 10^{-11}$ F

Energy of charged conductor is

$$U = \frac{q^2}{2C} = \frac{(5 \times 10^{-6})^2 \text{C}}{2 \times 2 \times 10^{-11} \text{ F}} = 0.625 \text{ J}$$

15. (i) (d): In polar molecule the centres of positive and negative charges are separated even when there is no external field. Such molecule have a permanent dipole moment. Ionic molecule like HCl is an example of polar molecule.

(ii) (c) : As $F_m = \dfrac{F_o}{K}$

$\therefore$ The maximum force decreases by K times.

(iii) (b)

(iv) (b): A polar molecule is one in which the centre of gravity for positive and negative charges are separated.

(v) (a)